adjusting μ_j, λ_j and $Z(0)$, $Z(1)$:

$$E_R[\lambda_i,\, \mu_i,\, Z(0),\, Z(1)] = \sum_{i=1}^{M} \left[\frac{\sin\left(\sqrt{\lambda_i}\right)}{\sqrt{\lambda_i}} \prod_{j=1}^{M} \frac{\mu_j - \lambda_i}{Q_j - \lambda_i} - (-1)^{i+1} \sqrt{\frac{Z_g Z_\ell}{Z(0)Z(1)}}\, \frac{g(\lambda_i)}{\sqrt{\lambda_i}} \right]^2 + \sum_{i=1}^{M} \left[\cos\left(\sqrt{\mu_i}\right) \prod_{j=1}^{M} \frac{\lambda_j - \mu_i}{P_j - \mu_i} - (-1)^i \sqrt{\frac{Z_g Z(1)}{Z(0) Z_\ell}}\, g(\mu_i) \right]^2 + \left[\prod_{j=1}^{M} \frac{\lambda_j}{P_j} - \sqrt{\frac{Z(1)}{Z(0)}} \right]^2. \tag{14}$$

The resultant μ_j and λ_j are used to generate Cauchy data, while the resultant $Z(0)$ and $Z(1)$ are used as boundary data to calculate $Z(x)$ from $q(x)$. The successive approximation method described in [8] failed to recover $q(x)$ from the Cauchy data. As is shown in Fig. 2(a), $q(x)$ diverges after two iterations. However, we can use the proposed method to calculate $q(x)$ without difficulty. For reference, Fig. 2(b) and (c) shows the related characteristic impedance and simulated $|S_{11}(j\omega)|$ of the NTL filter, respectively.

IV. Conclusions

When applied to constructing $q(x)$ from Cauchy data, the successive approximation method suffers from two disadvantages, i.e., several iterations are needed and they sometimes fail when the corresponding $q(x)$ is large and oscillates rapidly. In the present algorithm, $q(x)$ can be recovered straightforwardly from Cauchy data by using the exact data of $q(x)$ at $x = \ell$ and adopting a proper discrete form of the related integral equation; hence, the above disadvantages are avoided. The numerical error of the proposed method mainly depends upon sampling spacing Δ.

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The Reentrant Wide-Band Directional Filter

Anatoly Petrovich Gorbachev

***Abstract*—A new type of a wide-band microwave filter is described and named the reentrant directional filter, in which resonance occurs in the form of a traveling wave rather than in the conventional form of a standing wave. This device is the network, which has the constant input impedance and is manufactured as the directional coupler's free construction. An analysis of the reentrant directional filter shows it to have advantages in the case of wide-bands when compared to previously used directional filters. This filter finds application in multiplexers, as well as in matched bandpass (band-stop) filters by using planar multilayer transmission-line technology. Experimental results verify the theoretical approach.**

***Index Terms*—Directional filters, microwaves, multilayer, strip-line components, wide-band.**

I. Introduction

It is well known [1] that directional filters are one type of device capable of performing systems that use frequency-division multiplexing. For example, wide-band multiplexers have been analyzed and constructed as described in [2].

The basic directional filter proposed in [3] has been developed in many papers [4]–[6]. However, the above-mentioned directional filters are themselves badly adapted to multilayer (multilevel)-strip transmission-line technology, which has been an increased interest in recent years in RF integrated circuits. For instance, the analysis and design of multilayer coupled-line directional couplers has recently been reported in [7].

This paper present a novel wide-band directional filter called a "reentrant directional filter," which is better adapted itself to multilayer planar transmission-line technology through the full screening of its fragments.

II. Analysis

To better understand the above-mentioned directional filter, it should be analyzed on a base of a strip-coaxial model, although practical devices remain to be constructed in multilayer-strip transmission-line realization.

Fig. 1 shows the new reentrant directional filter (patent pending). Conductors A–D are coaxial-line center conductors of characteristic impedance Z_B and electrical length θ_B with relative dielectric constant ε_{rB} of the coaxial-line medium (brief expression—"transmission line (Z_B, θ_B)") within the loop L of electrical length $4\theta_N$. The loop L is formed from the center conductor of the transmission line of characteristic impedance Z_N with corresponding dimensions w_a, w_B, t, and b and relative dielectric constant ε_{rN} of the directional filter medium within ground conductor G. The arms of the conductors A–D are connected together by a narrow conducting link. The link is designed to be narrow so that there will be no propagation around the end of the center conductors, but rather, that the even and odd excitation will be terminated by open and short circuits, respectively. Distance a is chosen to avoid electromagnetic coupling between opposite edges of loop conductor L. The external arms of the conductors A–D are directional filter terminal ports 1–4, respectively.

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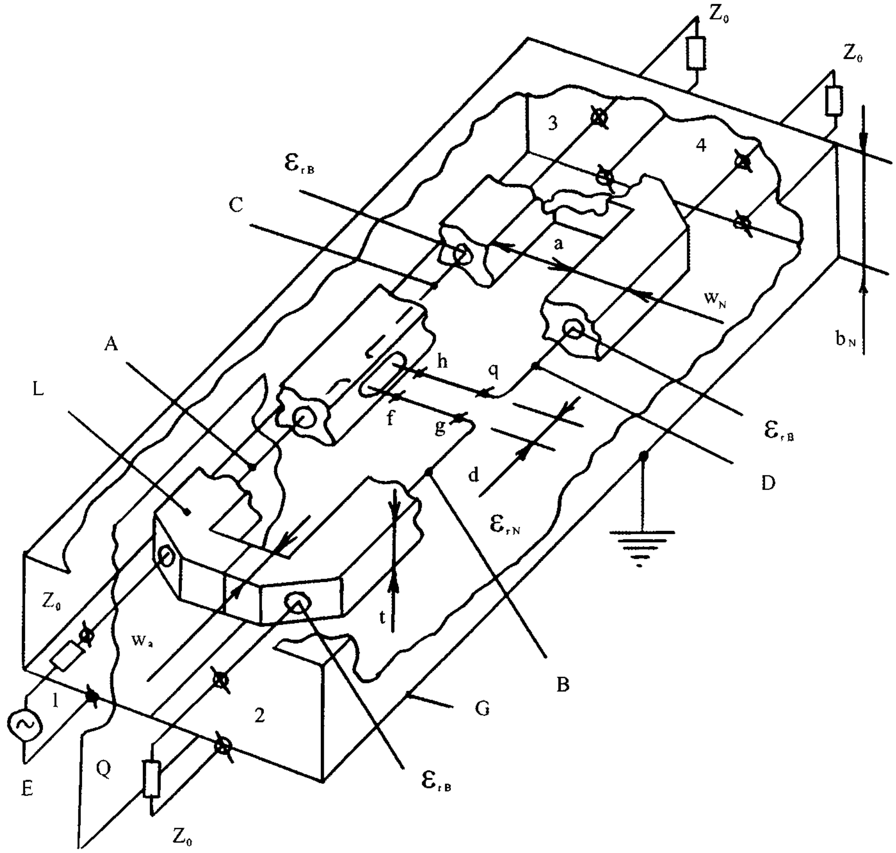

Fig. 1. View of reentrant directional filter.

The method of analysis used here follows the conventional approach developed by Reed and Wheeler [8] and has been adopted for the directional filter, whereby the total voltages and currents on the structure are obtained by applying the principal of superposition of the even- and odd-mode (excitation) solutions. The even-mode solution occurs when the structure is bisected through its vertical plane of symmetry Q by a magnetic wall, while the odd-mode solution occurs when the bisecting plane is an electric wall. Therefore, the configuration of the structure in Fig. 1 can be reduced to two equivalent even- and odd-mode circuits leading to the corresponding scattering matrices $[S_e]$ and $[S_o]$

$$[S_e] = \begin{bmatrix} S_{11e} & S_{12e} \\ S_{12e} & S_{11e} \end{bmatrix} \quad [S_o] = \begin{bmatrix} S_{11o} & S_{12o} \\ S_{12o} & S_{11o} \end{bmatrix}. \tag{1}$$

These matrices are defined in a widely used manner by using the general definition for scattering parameters [9] of the four-port circuits with the reentrant cross sections [10]. In this way, the overall scattering matrix $[S]$ for the symmetrical directional filter may be written as

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{11} & S_{14} & S_{13} \\ S_{13} & S_{14} & S_{11} & S_{12} \\ S_{14} & S_{13} & S_{12} & S_{11} \end{bmatrix} \tag{2}$$

$$S_{11} = (S_{11e} + S_{11o})/2$$
$$S_{12} = (S_{11e} - S_{11o})/2$$
$$S_{13} = (S_{12e} + S_{12o})/2$$
$$S_{14} = (S_{12e} - S_{12o})/2.$$

To calculate the frequency sensitivity of the device, the junctions are still assumed to be pure shunt or series connections. The frequency-dependent electrical lengths θ_B and θ_N in the matrices are expressed in terms of normalized frequencies f/f_{0B} and f/f_{0N}, respectively, as follows:

$$\theta_B = \left(\frac{\pi}{2}\right)\left(\frac{f}{f_{0B}}\right) \quad \theta_N = \left(\frac{\pi}{2}\right)\left(\frac{f}{f_{0N}}\right). \tag{3}$$

The values f_{0B} and f_{0N} are the distinctive frequencies of the corresponding fragments, i.e., if $f = f_{0B}(f = f_{0N})$, then $\theta_B = 90^\circ(\theta_N = 90^\circ)$. In general, $\theta_B \neq \theta_N$ because of the possible differences of mediums ($\varepsilon_{rB} \neq \varepsilon_{rN}$) or physical lengths. However, the difference of distinctive frequencies can be equal to zero ($f_{0B} = f_{0N}$) by subsequently choosing the appropriate mediums and/or physical lengths.

When four terminal ports of the structure illustrated in Fig. 1 are terminated in the proper constant resistance, the device is like a directional filter with constant input impedance ($S_{11} = 0$) and infinite directivity ($S_{14} = 0$) at all frequencies. In this filter, energy is transferred backward instead of forward. Hence, if a signal is fed into the first terminal port, part of that signal will emerge at terminal port 2 and the remainder at third terminal port. Furthermore, this energy division is frequency selective with no signal emergence from the diagonally opposite fourth terminal port.

The characteristics of this reentrant directional filter can be determined rigorously from the scattering matrix (2) by varying in magnitude the characteristic impedance Z_B and Z_N with those prescribed by the terminal transmission lines Z_0, which are bound to be equal to the constant resistance. The parameter-optimization problem was solved by means of one of the numerical methods of nonlinear programming [11]. The corresponding goal function is

$$F(\theta) = \sum_{i=1}^{N} \left[\left| S_{11}(\theta_i) \right| + \left| S_{14}(\theta_i) \right| \right]. \tag{4}$$

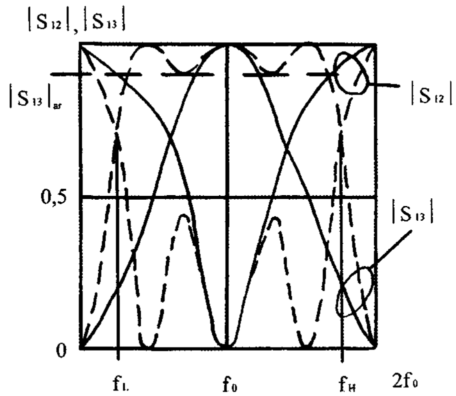

Fig. 2. Computed responses of the reentrant directional filters.

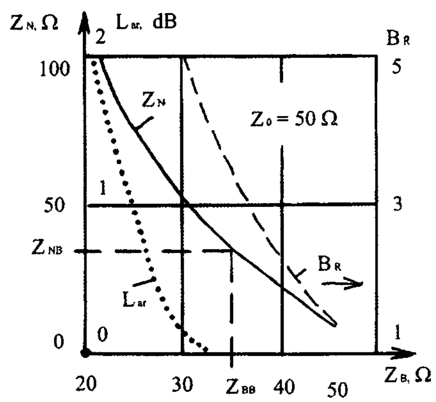

Fig. 3. Complete results of an optimization process of the reentrant directional filters.

Here, $\theta_i = \theta_{Bi} = \theta_{Ni} = (\pi/2)(f_i/f_0)$, f_0 is the center frequency of a frequency range $f_0 = f_{0B} = f_{0N}$. The frequencies $f_1, \ldots, f_{(N-1)/2}$ are placed on the left-hand side of f_0. The frequency $f_{(N+1)/2}$ is equal to the center frequency. The following frequencies f_i are placed on right-hand side of f_0, where N is odd. The frequency steps $\delta_f = f_{i+1} - f_i$ are independent with a number i and are chosen in the $(0.01, \ldots, 0.0001)f_0$ range.

If the goal function (4) reduces to a minimum with the frequency step $\delta_f = 0.01 f_0$ and $N = 41$, then the optimum value of the impedance Z_B and Z_N are determined. The characteristic impedance Z_0 is equal to 50 Ω. For all, the electrical lengths $\theta_a = 2\pi a/\lambda_0^\varepsilon$ and $\theta_d = 2\pi d/\lambda_0^\varepsilon$ ($\lambda_0^\varepsilon = 2.9979 \cdot 10^8/(f_0\sqrt{\varepsilon_{rN}})$) are negligible, in respect to those for coaxial line center and loop conductors. For example, they are

$$\theta_a = \theta_N/(10, \ldots, 15) \quad \theta_d = \theta_N/(12, \ldots, 17). \tag{5}$$

Thus, the optimum impedances Z_B and Z_N may be defined, which provide the directional filter's usual frequency responses. For instance, Fig. 2 shows the responses of two versions of these filters. The maximally flat version according to $Z_B = 40\ \Omega$ and $Z_N = 22.5\ \Omega$ is depicted by a solid line and the other corresponding to $Z_B = 25\ \Omega$ and $Z_N = 75\ \Omega$ is depicted by a dashed line. It may be seen that it is possible for the filters to perform with an equal-ripple response, while the familiar filters [3], [4], [6] with one loop conductor cannot be shaped from that. A border between the equal-ripple and maximally flat responses is the numerical solved impedance to be equal to

$$Z_{BB} = Z_{NB} = Z_0/\sqrt{2}. \tag{6}$$

The results of this optimization process are presented in Fig. 3 as the functions of impedance Z_B. Here, Z_N is a solid line, which mapped

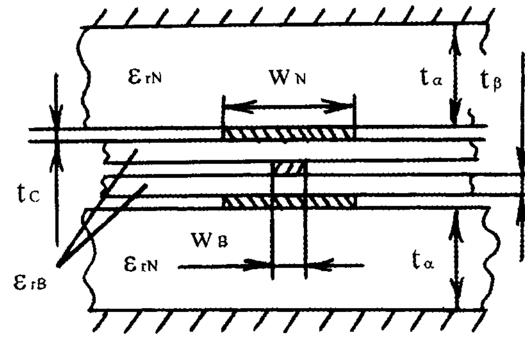

Fig. 4. Four-layer reentrant directional filter.

the loop conductor impedance. The value B_R is a dashed line denoted by the bandwidth ratio of a bandpass channel $1 \leftrightarrow 3$ by means of $1/\sqrt{2}$ (3 dB): $B_R = f_H/f_L$ (see Fig. 2). The attenuation $L_{ar} = 10\lg(1/|S_{13}|_{ar}^2)$ is a dotted line related to the ripple mean in a passband when the impedance Z_B is chosen less than Z_{BB}. Since the directional filter is symmetric with respect to two planes, it is evident that the two outputs at ports 2 and 3 differ in phase by 90° at all frequencies. Thus, $\varphi_{13} - \varphi_{12} = 90°$, $\varphi_{12} = \arg(S_{12})$, $\varphi_{13} = \arg(S_{13})$.

III. General Design Procedure and Experimental Results

In order to demonstrate the principles, the 66% bandwidth directional filter with a maximally flat response over an octave frequency range was synthesized. The center frequency is 1.5 GHz corresponding to $\theta_B = \theta_N = 90°$. From Fig. 3, for B_R equal to two, we have

$$Z_B = 41\ \Omega \quad Z_N = 20\ \Omega. \tag{7}$$

The printed-circuit version of this directional filter is manufactured in a four-layer Teflon strip-line technique with a commercially available thin-core copper-clad center boards made from Russian dielectric material "Φ4 MЂCΦ-2" and with ground-plane boards made from Russian dielectric material "ΦAΦ-4" unclad on one side and copper clad on the other. The thickness t_β and t_α of these boards are 0.27 and 1.5 mm, respectively. The thickness t_C of the copper peel is equal to 0.02 mm. The hose was Teflon because of its very low dissipation factor and small dielectric-constant variation, i.e., $\varepsilon_{rB} = \varepsilon_{rN} = 2.5$.

The dimensions of the lines are designed to produce the required values Z_B and Z_N according to (7). The impedance Z_N is determined from the impedance of a metallic bar of width w_N and height $t_N = 2t_\beta + 3t_C = 0.6$ mm placed within a ground-plane spacing $b_N = 2t_\alpha + t_N = 3.6$ mm, as shown in Fig. 4. The value Z_B is obtained from the impedance of a strip of thickness t_C and width w_B placed within a ground-plane spacing $b_B = 2t_\beta + t_C = 0.56$ mm. To determine the strip-line widths, the formulas in [1] served for calculation; thus, we have $w_B = 0.53$ mm and $w_N = 6.65$ mm.

In Fig. 5, in this version of the filter, two thin-core center boards of 1.5-mm thickness G_1 and G_2 are used , and are placed after following a reciprocal combination between the ground-plane boards. The center conductors A–D (0.53-mm width) have been etched on one side of the board G_1. On the other side, this board of the upper plane of the loop conductor L was realized; the lower plane of which has been etched on one side of the board G_2 with its other side being fully peel free. As a result, after assembling the filter, we have the following dimensions: $w_a = w_N = 6.65$ mm, $a = 2$ mm, and $d = 1.5$ mm. Furthermore, a required equipotential surface of both of the loop conductor planes can be obtained by a solder through 12 pieces of 0.4-mm-diameter metallized holes (MHs), as shown in Fig. 5. The connectors for the strip lines were four Russian "$\ni 2 - 116/1$" with the terminating strip-line width w_0 equal to 2.7 mm. The detailed information of Russian connectors

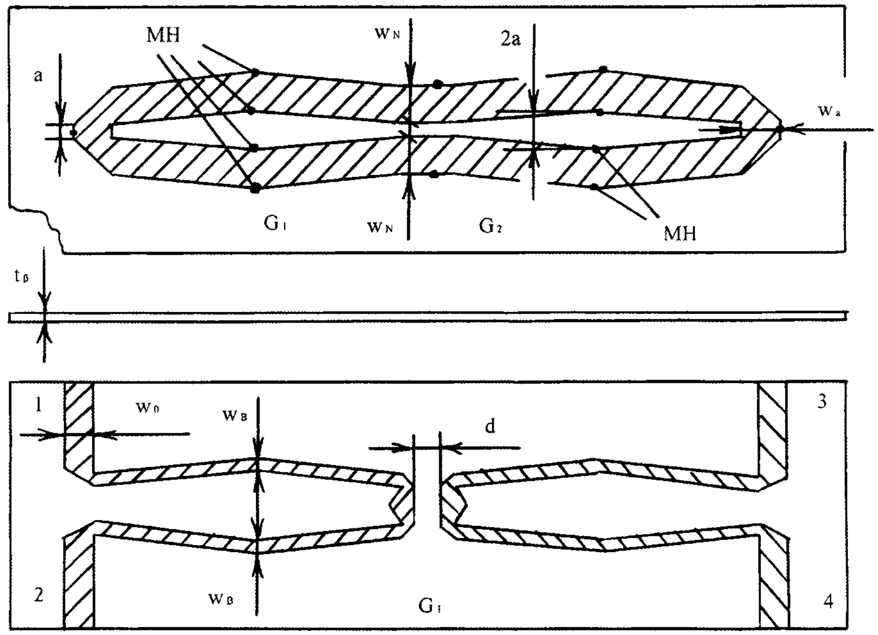

Fig. 5. First and second centered boards of the manufactured filter.

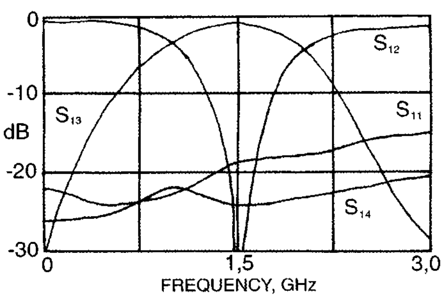

Fig. 6. Measured S-parameters of the reentrant directional filter.

and microwave materials has been published in Russian sources [12], [13].

The filter was tested on "P4-11" and "P4-38" Russian network analyzers, which confirmed a good agreement of experimental results with numerical solutions. Measured results for the scattering parameters shown in Fig. 6 were also satisfactory.

IV. Conclusion

As has been described, the novel reentrant directional filter presents to the microwave engineering a new type of multilayer circuit, which is relatively simple in realization and has acceptable frequency responses, return loss, and directivity. The full-wave analysis would be appropriate for achieving more accuracy in the special case when a thickness of the boards is very low and/or the operating frequencies are higher.

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